KEY

Ch. 5 Problems- Review

<u>In Chapter 5 we looked at 4 different types of problems:</u>

- Binomial Probability with counts (small sample)
- Binomial Probability with counts (large sample)
- Binomial Probability with proportions (large sample)
- Sample Means

Directions: First, distinguish which type of problem you are looking at. Then proceed with appropriate work and solution. Be sure to show **probability notation and calculator work.**

1. Given that 58% of all gold dealers believe next year will be a good one to speculate in South African gold coins, in a random sample of 150 dealers, what is the probability that between 55% and 60% ← $\hat{p}$ believe that it will be a good year to speculate?

check ✓ $n \cdot p = 87 \geq 10$ $n(1-p) = 63$

$n = 150$ $p = 0.58$ $P(0.55 < \hat{p} < 0.6) =$ normalcdf$(0.55, 0.60, 0.58, \sqrt{\frac{(0.58)(0.42)}{150}})$ $= 0.462$

2. The time necessary to mow a "regular" size lawn is normally distributed with a mean of 35 minutes and a standard deviation of 3 minutes.

$\mu = 35$ $\sigma = 3$ no check

   a. If **one** lawn mower is chosen at random, what is the probability that the student finished in less than 33 minutes?

$P(x < 33) =$ normalcdf$(-E99, 33, 35, 3) = 0.2525$

   b. The average time necessary for 15 randomly selected lawn mowing students is computed. What is the probability that the **average** time for the **sample** is less than 33 minutes?

$n = 15$

$\mu_{\bar{x}} = 35$

$\sigma_{\bar{x}} = 3/\sqrt{15}$

$P(\bar{x} < 33) =$ normalcdf$(-E99, 33, 35, 3/\sqrt{15})$ $= 0.00491$

3. A machine that manufactures a part for a car engine typically produces defective parts 5% of the time. If a random sample of 30 parts is taken, what is the probability...

check $n \cdot p = 1.5 \geq 10$ ✗ $n(1-p) =$

$n = 30$ $p = 0.05$

   a. That the sample will contain anywhere from 10 to 20 defective items (**inclusive**)?

$P(10 \leq X \leq 20) =$ binomcdf$(30, 0.05, 20) -$ binomcdf$(30, 0.05, 9)$ $= 1.162 \times 10^{-6}$

   b. What is the probability that **more than** 10 defective items are found?

$P(X > 10) = 1 - P(X \leq 10) = 1 -$ binomcdf$(30, 0.05, 10) = 1.097 \times 10^{-7}$

   c. What is the probability that **between** 6 and 12 defective items are found?

$P(6 < X < 12) = P(7 \leq X \leq 11) =$ binomcdf$(30, 0.05, 11) -$ binomcdf$(30, 0.05, 6)$ $= 5.735 \times 10^{-4}$

   d. What is the probability that 8 **or less** defective items are found?

$P(X \leq 8) =$ binomcdf$(30, 0.05, 8) = 0.99998$

   e. What is the probability that **at least** 7 but **less than** 11 defective items are found?

$P(7 \leq X < 11) =$ binomcdf$(30, 0.05, 10) -$ binomcdf$(30, 0.05, 6)$ $= 5.7338 \times 10^{-4}$

$P(X \leq k)$ binomcdf(n, p, k)

4. I have a binomial distribution of a random variable X with B(200, 0.45). What are the ~~mean and~~ standard deviation of X?

$\mu_X = 90$ $\quad \sigma_X = \sqrt{n \cdot p \cdot (1-p)} = 7.036$

5. A certain type of car tire is known to have a mean lifetime of 40,000 miles and standard deviation of 2,500 miles. Suppose a tire store looks at 400 of these tires (which were randomly selected).

normal ?? $n \geq 30$ $400 \geq 30$ ✓ $\Rightarrow$ normal

   a. What is the probability the average lifetime of these 400 tires is between 35,540 and 43,104 miles?

$\mu_{\bar{x}} = 40000$

$\sigma_{\bar{x}} = \frac{2500}{\sqrt{400}}$

$P(35540 \leq \bar{x} \leq 43104) = \text{normalcdf}\left(35540, 43104, 40{,}000, \frac{2500}{\sqrt{400}}\right) = 1$

   b. Why are we allowed to use the normal distribution?

because $n \geq 30$! $\Rightarrow$ normal sampling distr.

6. We take a survey of 120 adults about President Bush. The adults are asked to decide whether they think that the president is doing a good job or not (only yes or no answers). According to USA today 53% of adults that think he is doing a good job.

check $n \cdot p = 63.6$ $n(1-p) = 56.4$ ✓

$n = 120$

$p = 0.53$

check? ✓

$\mu_X = 63.6$

$\sigma_X = 5.467$

   a. What is the probability that between 60 and 70 adults think that the president is doing a good job?

$P(60 \leq X \leq 70) = \text{normalcdf}(60, 70, 63.6, 5.467) = 0.624$

   b. What is the probability that more than 55% of adults think that the president is doing a good job?

$\mu_{\hat{p}} = 0.53$

$\sigma_{\hat{p}} = \sqrt{\frac{(0.53)(0.47)}{120}}$

$P(\hat{p} > 0.55) = \text{normalcdf}\left(0.55, E99, 0.53, \sqrt{\frac{(0.53)(0.47)}{120}}\right)$
$= 0.3595$

7. A random variable X has the distribution N(56, 3.5) ~~and a~~ random variable Y has the distribution N(45, 4.2). We want to take a sample of 40 of random variable X and a sample of 60 of random variable Y. These two variables are independent of each other.

   a. What is the **distribution** for the average of the **sample** for X? $n = 40$

$\bar{X} \sim N\left(56, \frac{3.5}{\sqrt{40}}\right) = N(56, 0.5534)$

   b. What is the **distribution** for the average of the **sample** for Y? $n = 60$

$\bar{Y} \sim N\left(45, \frac{4.2}{\sqrt{60}}\right) = N(45, 0.5422)$

   c. What is the **distribution** for the average of the sample for X-Y *(distribution of $\bar{X}-\bar{Y}$)*?

$\mu_{\bar{X}-\bar{Y}} = 56 - 45 = 11$

$\sigma_{\bar{X}-\bar{Y}} \Rightarrow \sigma^2_{\bar{X}-\bar{Y}} = \sigma^2_{\bar{X}} + \sigma^2_{\bar{Y}} = (0.5534)^2 + (0.5422)^2 = 0.6002324$

$= \sqrt{0.6002324}$

$= 0.7747$

$N(11, 0.7747)$

   d. What is the probability that the average of X-Y in the sample is less than 10?

$P(\bar{X} - \bar{Y} < 10) = \text{normalcdf}(-E99, 10, 11, 0.7747)$
$= 0.0984$